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1966-21

Position Error  
in Station-Keeping Satellite

B. J. Moriarty

1 April 1966

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Lexington, Massachusetts



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MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
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POSITION ERROR IN STATION-KEEPING SATELLITE

*B. J. MORIARTY*

*Group 63*

TECHNICAL NOTE 1966-21

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## ABSTRACT

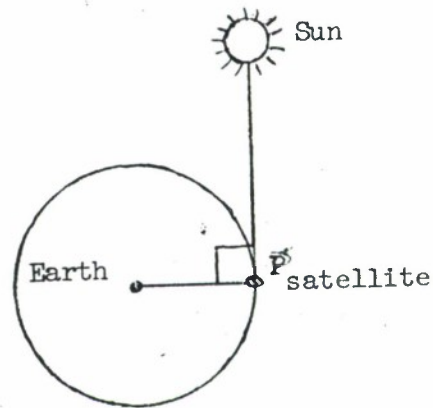
An automatic station-keeping satellite must be able to measure its deviation from the nominal position. For the system considered here (an equatorial synchronous orbit) this involves a latitude difference measurement between the earth and sun at certain times. This report shows that a combination of an error in satellite orientation and the sun being in a plane other than the equatorial plane may result in errors in position determination. The nature of the errors involved is described and their magnitudes estimated.

Accepted for the Air Force  
Franklin C. Hudson  
Chief, Lincoln Laboratory Office

## Position Error in Station-Keeping Satellite

### Ideal System Operation

The satellite is in a synchronous equatorial orbit and its spin axis is parallel to the orbit normal. It is equipped with sensors which measure the longitude difference in satellite coordinates between the sun and the earth. If the satellite is accurately synchronous, the longitude difference should be a certain value (say  $90^\circ$ ) at the same time (measured by an on-board clock) each day. Thus a measurement of the time at which the longitude difference is  $90^\circ$  may be used to control a position correcting mechanism.



### Statement of Problem

How will this system function when the satellite spin axis is not parallel to the orbit normal and the sun is not in the plane of the equator? That is, what errors may be expected in the estimation of longitude difference?

I first attempt rough estimates of the errors involved on an intuitive basis. A computer program, based on the vector approach, is described and results from this substantiate the numbers of the next section.

### Summary of this Note

Due to the sun's elevation an error  $\theta$  in satellite spin axis position may (depending on the details of the geometry) couple into an error in orbit position estimation of  $\theta \times .4$ . It appears at this date that an error of this magnitude is quite significant to functioning of the position control mechanism. This may indicate that a reduction of attitude error is required.

### Geometry of the Problem

1. First consider the problem when the sun S is in the equatorial plane of the earth as in Fig. 1 and the satellite axis A' makes an angle  $\theta$  with the orbit normal. Now E' S' is the longitude difference measured by the satellite. If the satellite were correctly oriented at this point in the orbit, it would measure the arc ES. Let OL be the line of nodes. Then from spherical geometry

$$\tan LE = \frac{\tan LE'}{\cos \theta}$$

$$\tan LS = \frac{\tan LS'}{\cos \theta}$$

$$ES = \tan^{-1} \left( \frac{\tan LS'}{\cos \theta} \right) + \tan^{-1} \left( \frac{\tan LE'}{\cos \theta} \right)$$

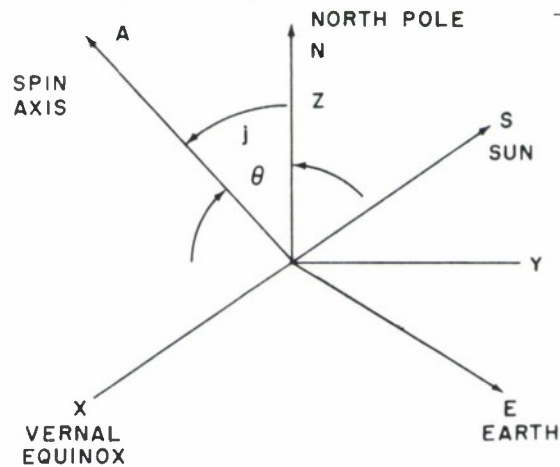
Thus for  $\theta < 10^\circ$  the error is less than  $1'$ .

2. Now consider the effect of the sun's declination on the problem. The previous paragraph has shown that with the sun in the earth's equatorial plane the satellite measures the arc E'S'  $\sim$  ES. Now let the sun be at its zenith  $S_1$ . Let the spin axis move from N to A': then for small tilt angles we may regard the sun longitude line A'S<sub>1</sub>S<sub>1</sub>' as hinging at S<sub>1</sub>. In order to estimate this lever arm effect we project onto a plane perpendicular to the line OS<sub>1</sub>. By the arguments of the previous paragraph the error in longitude estimation is SS<sub>2</sub>  $\sim$  S<sub>1</sub>S'. We find SS<sub>2</sub>  $\sim .4 \times \theta$  for the worst case. The worst case is when the right ascensions of the sun and spin axis differ by odd multiples of  $90^\circ$ . The error approaches zero when the right ascensions differ by even multiples of  $90^\circ$ .

### Computer Program Results

These are plotted on Figs. 2, 3, and 4. On each graph there are 2 curves. One curve is for the sun at zenith and the other for the sun is the earth's equator. The position of the sun is fixed for each curve. The longitude difference between earth and sun as seen by a correctly oriented satellite at that point at which a tilted satellite measures this difference as  $90^\circ$  is plotted as ordinate. The curve is now obtained by letting the spin axis move around the North Pole at a constant angle  $\theta$  and calculating this longitude difference at each point. The graphs are for  $\theta = 20^\circ, 1^\circ, 3^\circ$  respectively.

### Vector Solution of the Problem



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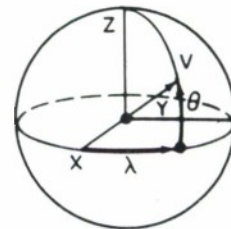
Given: The right ascensions and declinations of the spin axis A, the sun S, the longitude difference in satellite coordinates between the sun and earth.

Required: To find the longitude difference that would be measured by a correctly oriented satellite at this point.

Compute coordinates of unit vectors A, S from the right ascensions and declinations.

The X, Y, Z components of a unit vector V may be obtained from the right ascension and declination of

$$\begin{aligned} V_1 &= \cos \theta \cos \lambda \\ V_2 &= \cos \theta \sin \lambda \\ V_3 &= \sin \theta \end{aligned}$$



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We obtain components for the two vectors S and A.

We now obtain the position of E given the positions of S and A and that the longitude differences in satellite coordinates between S and E is  $\phi$ .

The vector  $A \times E$  is in the equatorial plane of the satellite, at right angles to the plane containing A and E. Let AE be the angle between A and E. Thus  $A \times E = \mu_E \sin AE$  where  $\mu_E$  is a unit vector.

Similarly  $A \times S = \mu_S |A \times S|$ .

Now a positive rotation  $\phi_A$  about A will bring  $\mu_E$  into coincidence with  $\mu_S$ .

$$\therefore \mu_E \times \mu_S = \sin \phi_A A$$

We also have

$$\mu_E \cdot \mu_S = \cos \phi_A$$

But

$$\mu_S \times (\mu_E \times \mu_S) = \mu_E - (\mu_E \cdot \mu_S) \mu_S$$

or

$$\mu_E = \mu_S \times A \sin \phi_A + \cos \phi_A \mu_S$$

This equation may be solved to give the components of  $\mu_E$ . E is the vector which fulfills the following conditions.

1.  $E \cdot \mu_E = 0$
2.  $E_3 = 0$  (the satellite is in an equatorial orbit)
3.  $|E| = 1$
4.  $A \times E = \mu_E \sin AE$  when  $AE < 180^\circ$



We obtain

$$E_1 = \frac{A_3}{|A_3|} \cdot \frac{\mu_{E2}}{\sqrt{(\mu_{E2})^2 + (\mu_{E1})^2}}$$

$$E_2 = -\frac{A_3}{|A_3|} \cdot \frac{\mu_{E1}}{\sqrt{(\mu_{E2})^2 + (\mu_{E1})^2}}$$

We now calculate the right ascension  $\lambda_E$  of the earth.

$$\cos \lambda_E = E_1$$

$$\sin \lambda_E = E_2$$

$$\therefore \tan \frac{\lambda_E}{2} = \frac{\sin \lambda_E}{1 + \cos \lambda_E} = \frac{E_2}{1 + E_1}$$

Let  $\lambda_S$  be the right ascension of the sun. Then at this point in the orbit, when the tilted satellite measured a longitude difference between the earth and the sun of  $90^\circ$  a correctly oriented satellite would measure  $\lambda_S - \lambda_E$ .

#### Acknowledgement

My thanks to Marie Roberts who implemented the program in a remarkably speedy fashion.

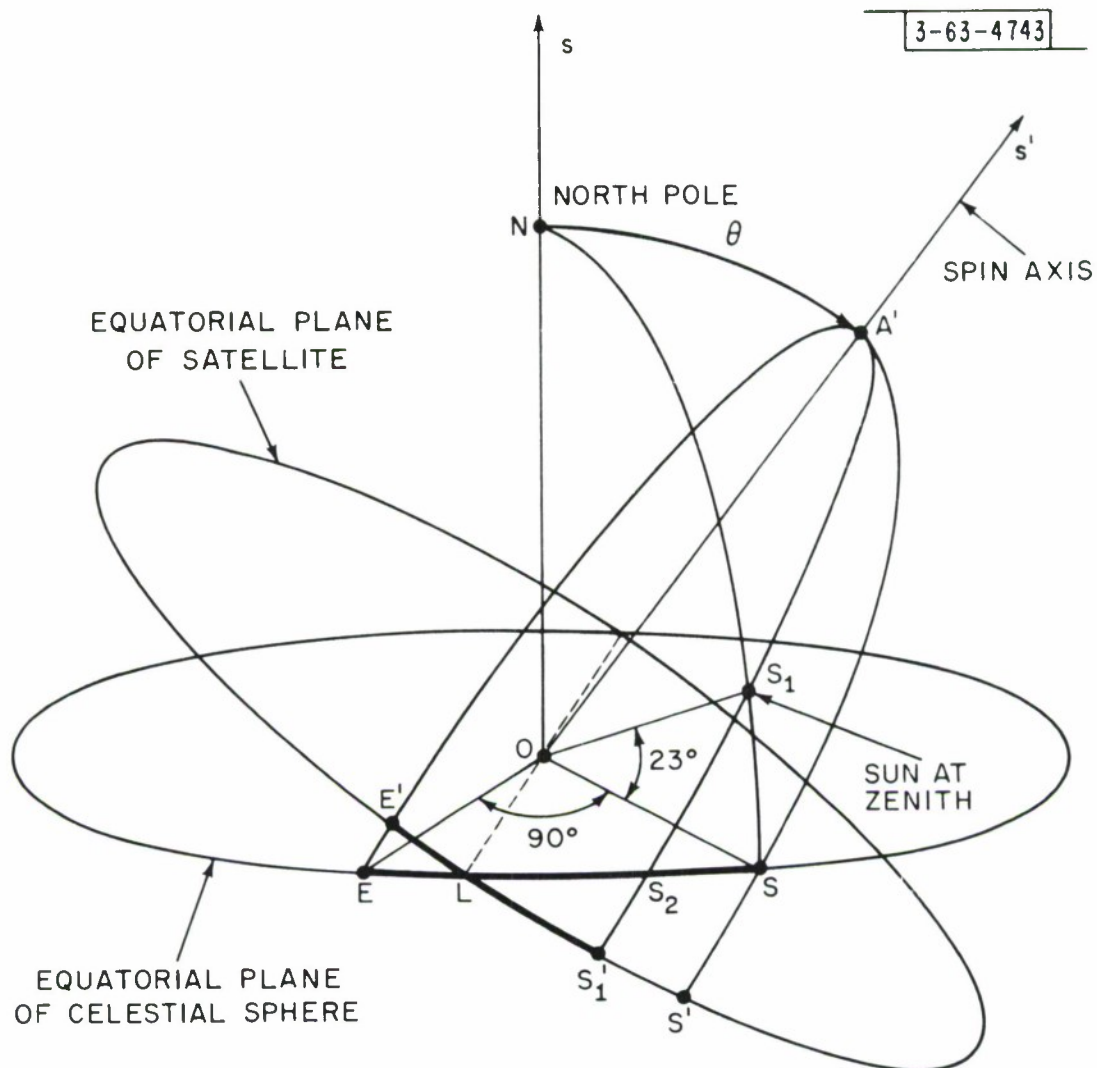


Fig. 1. Correctly oriented satellite (spin axis at A) measures arc ES ( $= 90^\circ$ ). With sun at S the satellite measures  $E'S' \sim ES$ . With sun at  $S_1$  the satellite measures  $E'S'_1 \sim ES - .4 \times \theta$ .

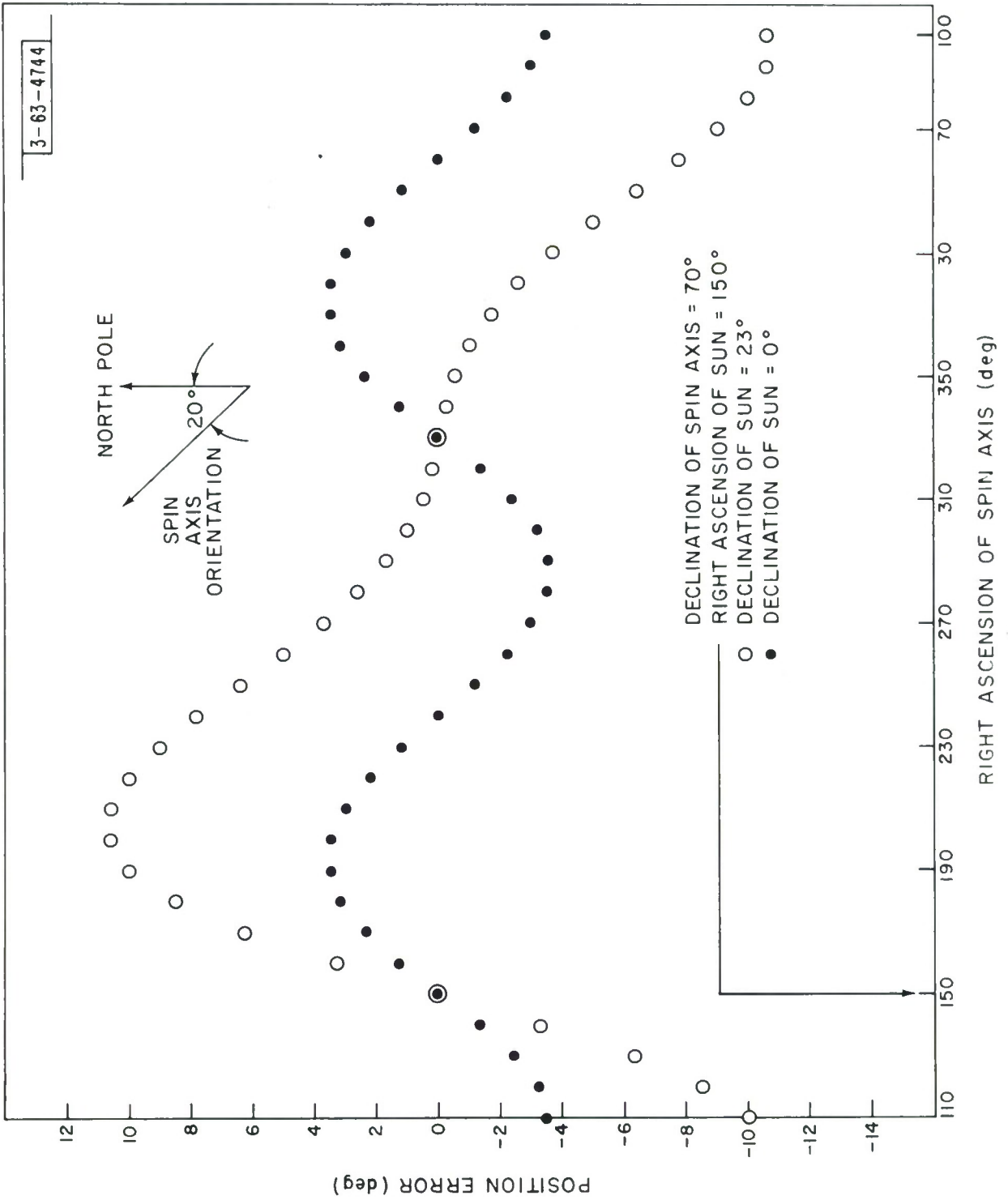


Fig. 2



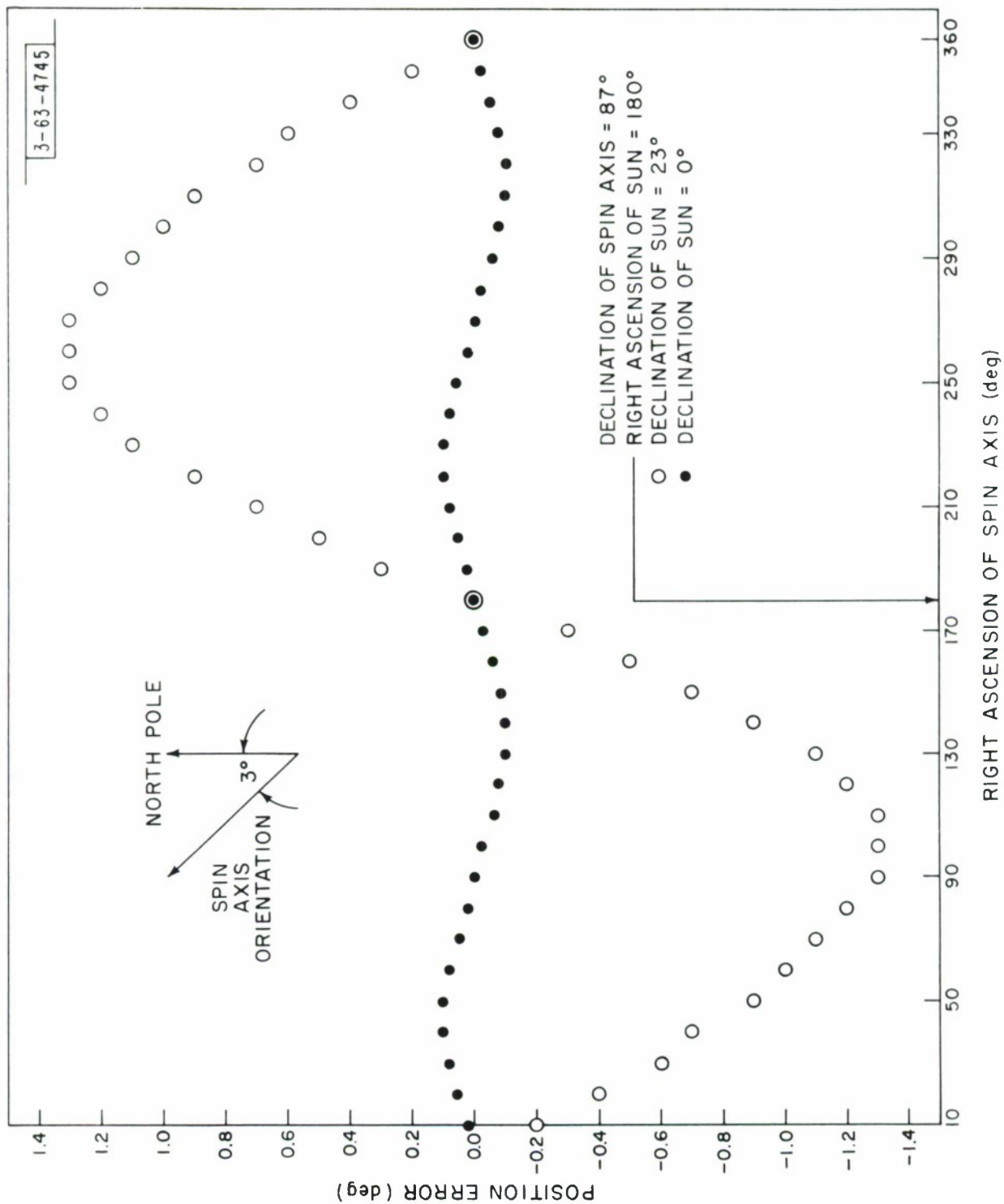


Fig. 3

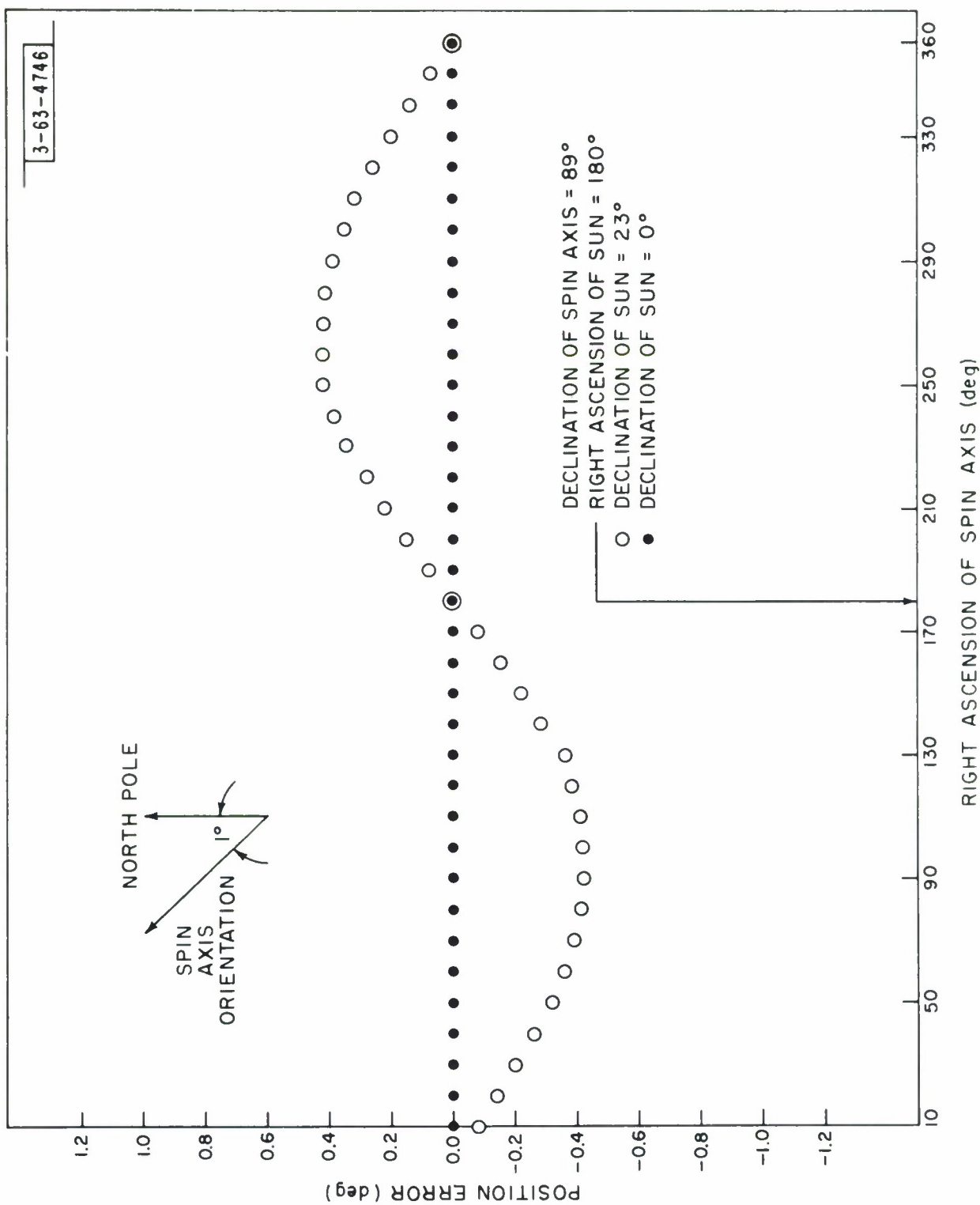


Fig. 4

## APPENDIX I

### Computation of Position Error

#### 1. Input

- (a) Right ascension and declination of the spin axis A(RA, DA)
- (b) Right ascension and declination of sun S(RS, DS)
- (c) Earth-sun longitude difference LES PRINT RA, DA, RS, DS

#### 2. Compute components of A along the axis

$$\begin{aligned}A1 &= \cos DA * \cos RA \\A2 &= \cos DA * \sin RA \\A3 &= \sin DA\end{aligned}$$

#### 3. Compute components of S along the axes

$$\begin{aligned}S1 &= \cos DS * \cos RS \\S2 &= \cos DS * \sin RS \\S3 &= \sin DS \\ \text{PRINT } A1, A2, A3, S1, S2, S3\end{aligned}$$

#### 4. Compute vector A x S

$$\begin{aligned}AS1 &= A2 * S3 - S2 * A3 \\AS2 &= A3 * S1 - S3 * A1 \\AS3 &= A1 * S2 - S1 * A2\end{aligned}$$

#### 5. Compute unit vector US

$$\begin{aligned}\text{Form } SQAS &= ((AS1)^2 + (AS2)^2 + (AS3)^2)^{1/2} \\US1 &= \frac{AS1}{SQAS} \\US2 &= \frac{AS2}{SQAS}\end{aligned}$$



$$US3 = \frac{AS3}{SQAS}$$

PRINT AS1, AS2, AS3

US1, US2, US3

Calculate  $MAGUS = ((US1)^2 + (US2)^2 + (US3)^2)^{1/2}$  and PRINT MAGUS

6. Compute unit vector UE

$$UE1 = (US2*A3 - A2*US3)*\sin LES + US1*\cos LES$$

$$UE2 = (US3*A1 - A3*US1)*\sin LES + US2*\cos LES$$

$$UE3 = (US1*A1 - A1*US2)*\sin LES + US3*\cos LES$$

PRINT UE1, UE2, UE3

and  $\sqrt{(UE1)^2 + (UE2)^2 + (UE3)^2}$

7. Compute vector E

$$E1 = \frac{+ UE2}{+\sqrt{(UE2)^2 + (UE1)^2}} \times \frac{|A_3|}{A_3}$$

$$E2 = \frac{- UE1}{+\sqrt{(UE2)^2 + (UE1)^2}} \times \frac{|A_3|}{A_3}$$

PRINT E1, E2 and  $\sqrt{(UE2)^2 + (UE1)^2}$

8. Compute right ascension of the earth

$$RE = 2 \tan^{-1} \left( \frac{E2}{1 + E1} \right) + 2\pi, \text{ mod } 2\pi$$

convert to degrees and PRINT

9. Compute

$$X = RS - RE + 360, \text{ mod } 360$$

PRINT R.A. Diff. = X

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